Influence of Boundary-Layer Stability on the Magnus Effect

IRA D. JACOBSON* AND J. B. MORTON*
University of Virginia, Charlottesville, Va.

Theme

THE stability of the boundary layer on a yawed spinning cylinder is analyzed. Both approximate analytic and numerical solutions are developed which agree well. Calculations for the Magnus force and moment on partially turbulent bodies are carried out giving new insight into the Magnus effect.

Contents

For a semi-infinite, yawed, spinning cylinder in an incompressible, viscous flow the laminar boundary-layer velocity profiles are given by Martin.[1] These solutions can be written as series expansions in terms of the reduced angle of attack $\alpha^* = \alpha x/R$ where α is the angle of attack, R the radius and x the distance from the leading edge; and, reduced Rossby number $\omega^* = V_\infty/\omega x$ where V_∞ is the freestream velocity and ω the spin rate.

For the purpose of the stability analysis only terms of first order in α^* and ω^{*-1} are retained. The boundary-layer thickness has been assumed small with respect to the body radius and the parallel flow assumption has been made.

Using the usual small perturbation approach the unsteady Navier-Stokes equations can be reduced to an Orr-Sommerfeld equation of the form

$$v'''' - 2\alpha_1{}^2 v'' + \alpha_1{}^4 v = iRe_\delta \gamma[(Q_1{}^* - \bar{C}_1)(v'' - \alpha_1{}^2 v) - Q_1{}^{*\prime\prime} v] \quad (1)$$

where v is the velocity perturbation in the y direction (see Fig. 1) and Re_δ is the Reynolds number based on the freestream velocity and the boundary-layer thickness, δ, $\bar{C}_1$ is the wave speed divided by γ and the $'$ denotes differentiation with respect to y/δ. The wave vector $\bar{k} = (\gamma, \beta, 0)$ enters the equation through $\alpha_1 = |\bar{k}|$ and $Q_1{}^* = \bar{k} \cdot \bar{U}/\gamma V_\infty$ where $\bar{U}$ is the steady-state laminar boundary-layer velocity profile.

The function $Q_1{}^*$ is not in general monotonic and for some values of β/γ exhibits overshoot and/or inflection points. Because of this, the analytic technique developed by Lin[2] for solving the Orr-Sommerfeld equation does not necessarily apply. This

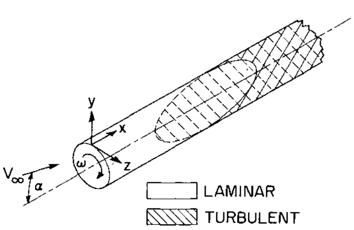

Fig. 1 Geometry.

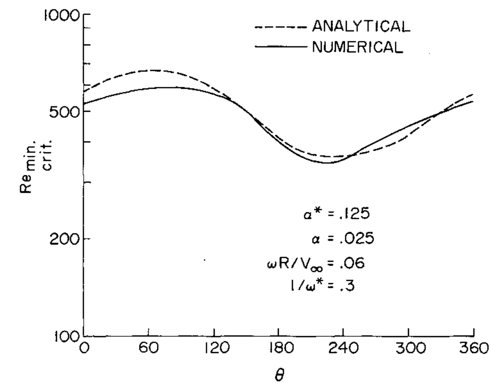

Fig. 2 Minimum critical Re vs azimuthal position.

question has been investigated by solving Eq. (1) numerically.† For a typical case the minimum critical Reynolds number vs azimuthal position is shown in Fig. 2.

These results suggest that an approximate analytic solution (such as that developed by Lin) with $\beta/\gamma = -(\omega R/V_\infty)$ is appropriate. An analysis based on this approach yields for the minimum critical Reynolds number

$$Re_{\delta\,\substack{\text{minimum}\\\text{critical}}} = \frac{43.1}{\bar{C}_1{}^4}\left[1 + \left(\frac{\omega R}{V_\infty}\right)\right] \times \left[\frac{0.3321[1+(\omega R/V_\infty)] + \alpha_l{}^*\varepsilon'(0)}{[1+(\omega R/V_\infty)^2 - 2\alpha(\omega R/V_\infty)\sin(z/R)]^2}\right] \quad (2)$$

where $\bar{C}_1$, the wave speed at the critical height and $\varepsilon'(0)$ are given in Ref. 3. As can be seen in Fig. 2 this approximate solution yields gratifying results. In addition, Fig. 3 illustrates the dependence of the minimum critical Reynolds number on the two parameters α^* and $1/\omega^*$ for $\omega R/V = 0.06$. These curves remain essentially unchanged for other small values of $\omega R/V$. The introduction of an angle of attack makes the boundary layer more susceptible to the effects of spin. It should be noted here that for the larger vaues of α^* and $1/\omega^*$ this data should be considered only qualitative in nature due to the assumptions made.

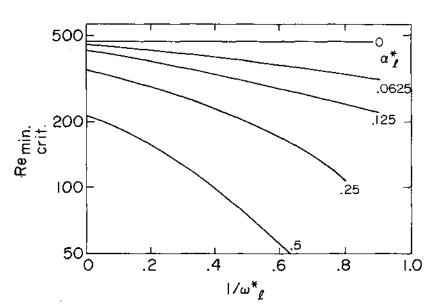

Fig. 3 Minimum critical Re vs α^* and $1/\omega^*$ for $\omega R/V = 0.06$.

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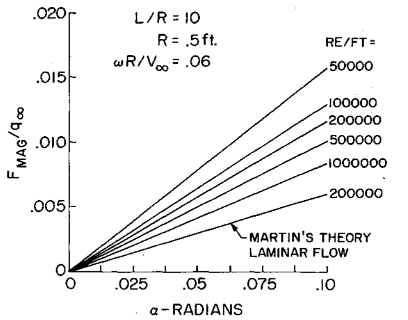
* Assistant Professor, Department of Aerospace Engineering and Engineering Physics. Member AIAA.

† A detailed discussion can be found in the back-up paper.

Fig. 4 Magnus force vs angle of attack.



The Magnus effect: The "small disturbance" analysis suggests that the transition of the boundary from laminar to turbulent flow occurs asymmetrically which would result in an asymmetric growth of the boundary layer (Fig. 1). The Magnus effect resulting from considering the potential flow about a "new" body, whose effective shape includes the displacement thickness, can be determined using slender body theory.

The boundary-layer displacement thicknesses grow at different rates for laminar[1] and turbulent[3] flows. For the turbulent case the growth was assumed to follow that of a boundary layer on a flat plate. Although not entirely adequate it was felt that this was sufficient to give a qualitative indication of the effect of the skewed transition line.

The transition from laminar to turbulent flow is assumed to take place at a Reynolds number proportional to the minimum critical Reynolds number. This is not strictly correct since in actual practice the transition depends on many factors (e.g., free-stream turbulence level, surface roughness, etc). However, this will tend only to shift the results in terms of Reynolds number without affecting the qualitative behavior.

Laplace's equation with the boundary conditions derived from the displacement thicknesses described previously yields a solution for the perturbed potential which is then used to calculate the pressure coefficient. Integrating to obtain the Magnus force and moment yields, respectively,

$$F_{\text{mag}} = -2R^2 q_\infty I(L)$$

and

$$M_{\text{mag}} = -2R^2 q_\infty \left[I(L)L - \int_0^{2\pi} \Delta(L) \sin\left(\frac{z}{R}\right) d\left(\frac{z}{R}\right) \right]$$

where q_∞ is the freestream dynamic pressure, Δ the boundary-layer displacement thickness, and

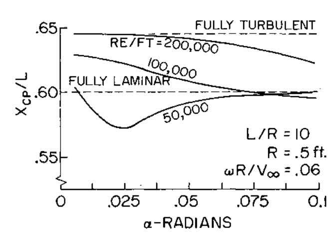
$$I = \int_0^{2\pi} \frac{\partial \Delta}{\partial x} \sin\left(\frac{z}{R}\right) d\left(\frac{z}{R}\right)$$

For the cases of fully laminar or fully turbulent boundary-layer flow, these can be integrated exactly to yield the same results as Martin.[1] For the more general case, the analysis is carried out numerically. The Magnus force predicted with an asymmetric mixed boundary layer is somewhat greater than that predicted with Martin's theory (Fig. 4).

The most striking result is in the behavior of the center of pressure of the Magnus force (Fig. 5). The center of pressure predicted on the basis of an asymmetric transition can be well forward of the fully laminar case. Due to the uncertainties in the parameters which determine the actual transition Reynolds number direct correlation with experimental data is not possible; the trends, however, agree quite well.

These results can only be used as an indication of the effect Reynolds number variation has on the Magnus center of pressure. If the actual transition Reynolds number based on body length is, for example, 200,000 at zero spin and zero angle of attack (as opposed to the assumed 80,000 above), then the behavior of the center of pressure for a given freestream Reynolds number and angle of attack changes. A very similar behavior is observed as in the previous case—the primary difference being the magnitude of the freestream Reynolds number for each curve.

Fig. 5 Center of pressure of Magnus force vs angle of attack.



References

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[2] Lin, C. C. "On the Stability of 2-D Parallel Flows, Parts I, II, III," *Quarterly of Applied Mathematics*, Vol. 1–3, 1943–1946.

[3] Jacobson, I. D. "Influence of Boundary Layer Transition on the Magnus Effect," Ph.D. dissertation, 1970, Univ. of Virginia, Charlottesville, Va.